A Theory and Method for Combining Multiple Approaches for Product Customization

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Abstract: Product design techniques that support mass customization include product platform design, computer-aided parametric design and modular design, among others. Unfortunately, no rigorous method has appeared until now to help designers combine these approaches systematically and maximize the quality and economic benefits of component commonality, particularly for very large number of product variants. In this paper we describe one method that addresses these issues, based on the formulation of the product design of customizable products as a problem of access in a geometric space. The method is illustrated with a case study, namely, the design of a line of customizable hand exercisers.

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2 Drs. Farrokh Mistree and Janet K. Allen gratefully acknowledge NSF Grant DMI-0085136.
Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_w$</td>
<td>Cost per kg</td>
</tr>
<tr>
<td>$S_e$</td>
<td>Endurance strength of wire material</td>
</tr>
<tr>
<td>$C$</td>
<td>Total annual cost</td>
</tr>
<tr>
<td>$S_u$</td>
<td>Ultimate tensile strength of wire material</td>
</tr>
<tr>
<td>$C_{inv}$</td>
<td>Inventory annual cost</td>
</tr>
<tr>
<td>$S_y$</td>
<td>Yield strength of wire material</td>
</tr>
<tr>
<td>$C_{mat}$</td>
<td>Material annual cost</td>
</tr>
<tr>
<td>$W$</td>
<td>Grip width</td>
</tr>
<tr>
<td>$C_{tool}$</td>
<td>Tooling annual cost</td>
</tr>
<tr>
<td>$Y$</td>
<td>Ideal production of springs per mandrel</td>
</tr>
<tr>
<td>$d$</td>
<td>Diameter of wire</td>
</tr>
<tr>
<td>$Z$</td>
<td>Expected number of springs produced in a mandrel before replacement</td>
</tr>
<tr>
<td>$D$</td>
<td>Spring coil diameter Greek</td>
</tr>
<tr>
<td>$\Delta F_1$</td>
<td>1st Space Division</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Demand of springs</td>
</tr>
<tr>
<td>$\Delta F_2$</td>
<td>2nd Space Division</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Mean (of a distribution of normal distribution of demand)</td>
</tr>
<tr>
<td>$\Delta F_3$</td>
<td>3rd Space Division</td>
</tr>
<tr>
<td>$\mu_{ln}$</td>
<td>Log-Mean (of a log-normal distribution of demand)</td>
</tr>
<tr>
<td>$E$</td>
<td>Elastic modulus</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Density of wire material</td>
</tr>
<tr>
<td>$F$</td>
<td>Grip force</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Tensile stress, Std deviation of demand</td>
</tr>
<tr>
<td>$h$</td>
<td>Grip height</td>
</tr>
<tr>
<td>$\sigma_{ln}$</td>
<td>Log-standard deviation (of a log-normal distribution of demand)</td>
</tr>
<tr>
<td>$H$</td>
<td>Height of spring</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Angular opening of torsion spring</td>
</tr>
<tr>
<td>$L$</td>
<td>Length of wire used in spring Sub-index</td>
</tr>
<tr>
<td>$K$</td>
<td>Spring coefficient</td>
</tr>
<tr>
<td>$D$</td>
<td>Mandrel, Spring coil diameter</td>
</tr>
<tr>
<td>$K_w$</td>
<td>Extra overhead cost for each wire utilized</td>
</tr>
<tr>
<td>$i$</td>
<td>1st space division</td>
</tr>
<tr>
<td>$k$</td>
<td>Ratio $d/D$</td>
</tr>
<tr>
<td>$j$</td>
<td>2nd space division</td>
</tr>
<tr>
<td>$m$</td>
<td>Number of mandrels</td>
</tr>
<tr>
<td>$k$</td>
<td>3rd space division</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of turns of spring coil</td>
</tr>
<tr>
<td>$W$</td>
<td>Wire</td>
</tr>
<tr>
<td>$R$</td>
<td>Normal distance from spring coil center to the center of applied grip force</td>
</tr>
</tbody>
</table>

1. Approaches to Product Design for Mass Customization

One of the key elements of mass customization is an adequate product design that balances product adaptability and component commonality. In trying to balance these two competing objectives, companies can combine multiple approaches for product customization. For example, Anderson (1997) describes the case of Matsushita’s customizable bicycles in which modularity is applied in wheels, pedals, drive chain, seats, handlebars and controls; adjustable features are applied to customize the height and angle of the seat and handlebars; and parametric CAD/CAM is utilized to tailor the lengths of the frame tubes. Typically, such a combination of approaches is made in an ad hoc manner. The lack of a rigorous and systematic method to combine multiple approaches is costly both in time and resources.

We utilize principles derived from "constructal" theory (Bejan, 2000) to develop a method that allows us to combine systematically multiple approaches for product customization.
The theoretic underpinning is described in Section 2. The various steps of the method are then described in Section 3. The method is illustrated with a case study, namely, the design of a customizable hand exerciser in Section 4, followed by closing remarks in Section 5.

2. A Theory for Product Design for Mass Customization

The starting point of the approach is modeling the set of all feasible combinations of values of product specifications that a manufacturing enterprise is willing to satisfy as a geometric space, which we call a space of customization. In this model, a product with one particular combination of specifications is represented as a point in this geometric space. In addition, we refer to any generic approach embedded in a product design for achieving systematically product customizations (such as parametric design, etc.) as a mode for managing product variety. The modes for managing product variety can be seen as the “vehicles” by which we “traverse” the space of customization as illustrated in Figure 1.

![Figure 1 -- Illustration of a Space of Customization](image)

Using the concept of a space of customization, the problem of designing customizable products using multiple modes for managing product variety can be formulated as a mathematical optimization problem: how to connect in the most effective manner all the points of a space of customization by combining multiple modes for managing product variety?

Problems of this kind are referred in the literature as access optimization problems, which are typically very difficult to solve. However, it has been recently proposed in constructal theory (Bejan, 2000) that such access problems can be solved efficaciously by solving the access in the space in a hierarchic manner through the sequential optimization of geometric shape at various scale levels, as shown in Figure 2.
Following the propositions of constructal theory, we solve our access problem through the hierarchic application of modes for managing product variety. We formulate the resulting problem mathematically as follows:

<table>
<thead>
<tr>
<th>Given</th>
<th>An N-dimensional space of customization $M^N = {r_1, r_2, ..., r_N}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$n$ Modes for Managing Product Variety</td>
</tr>
<tr>
<td></td>
<td>$k$ Stages</td>
</tr>
<tr>
<td>Find</td>
<td>The decision variables for each stage $\Delta r(i), \Delta r(2), ..., \Delta r(k)$</td>
</tr>
<tr>
<td></td>
<td>where $\Delta r(i) = [\Delta r_1(i), \Delta r_2(i), ..., \Delta r_N(i)]$</td>
</tr>
<tr>
<td>Satisfy</td>
<td>Constraints</td>
</tr>
<tr>
<td></td>
<td>$\Delta r_j(i+1) \geq \Delta r_j(i)$ for $j=1, ..., N$; $i=1, ..., k-1$</td>
</tr>
<tr>
<td></td>
<td>Bounds</td>
</tr>
<tr>
<td></td>
<td>$\Delta r_{j\min} \leq \Delta r_j(i) \leq \Delta r_{j\max}$</td>
</tr>
<tr>
<td>Optimize</td>
<td>An objective function $f$</td>
</tr>
</tbody>
</table>

In the preceding formulation, $\Delta r(i)$ represents a vector of $N$ decision variables that define the organization of the space of customization (the size and shape of the space elements) at stage $i$:

$$\Delta r(i) = [\Delta r_1(i), \Delta r_2(i), ..., \Delta r_N(i)]$$

The constraint $\Delta r_j(i+1) \geq \Delta r_j(i)$ in this formulation is required to enforce the solution to be a hierarchic construct, as described in constructal theory. Such a hierarchic organization of the space of customization has a number of advantages. For instance, as argued by Nobel Laureate and economist Herbert Simon in *The Sciences of the Artificial* (Simon, 1996), hierarchic structure enables systems to adapt and respond efficaciously to changes in the environment of different natures and scales. Since adaptability and fast responsiveness are foundational elements of mass customization, it follows that hierarchic organization of the modes for managing product variety, as proposed in this paper, should be considered an important property of product design for mass customization.
Particular favorable conditions for a hierarchic organization exist when each branch of the hierarchy operates independently of other branches. In other words, to take better advantage of hierarchic organization, the synthesis and evolution of one branch of a hierarchic construct should be independent of the synthesis and evolution of other branches. We will discuss more the implications of this property in the case example of Section 4.

In general, the combination of multiple modes for managing product variety is based on three principles proposed by Hernandez (2001):

1. Available modes for managing product customization should be utilized hierarchically in the design of product platforms for customizable products.
2. A hierarchy of modes for managing product customization should be organized in such a way that it provides easier access from a technology base (a platform of features, components and processes) to any possible set of specifications within the bounded space of customization to be serviced.
3. The optimal access for a given bounded space of customization can be obtained by optimizing the system access at every dimension scale. The solution should be constructed in a sequence that begins with the modes for managing product variety that deal better with high frequency and short-scope changes, and proceeds toward connecting these modes with shorter-frequency and larger-scope changes.

Based on the preceding principles, we describe in Section 3 a six-step method to design products that combine multiple approaches for product customization.

3. A Constructal Theory to Product Design for Mass Customization

Building upon the theoretic underpinning described in the preceding section, we propose to solve the design of products for mass customization as a problem of access in a geometric space with the six steps shown in Figure 3.

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Define the Space of Customization</td>
</tr>
<tr>
<td>2</td>
<td>Formulate an Objective</td>
</tr>
<tr>
<td>3</td>
<td>Identify Modes for Managing Product Variety</td>
</tr>
<tr>
<td>4</td>
<td>Determine Number of Hierarchy Levels and Allocate the Modes for Managing Product Variety to These Levels</td>
</tr>
<tr>
<td>5</td>
<td>Formulate a Multi-Stage Optimization Problem</td>
</tr>
<tr>
<td>6</td>
<td>Solve the Optimization Problem</td>
</tr>
</tbody>
</table>

Figure 3 -- Six-Step Method to Design Products that Combine Multiple Approaches for Product Customization

In the first step of the method, the specifications and boundaries of the space of customization are defined. For example, assume a product is customized in three specifications, say $x$, $y$ and $z$. The associated space of customization is three-dimensional. Once the parameters that define the space of customization are known, the allowable
range of values of each of these parameters are set. If no other constraints are considered (i.e., if all combinations of these variables are allowed) and $x$, $y$, and $z$ are all continuous, then the space of customization is as shown in Figure 4a. Note that a space of customization is not necessarily, or actually almost never, continuous. For example, the company could, instead offer discrete values of $z$, thus creating a mixed continuous-discrete space of customization, as shown in Figure 4b. Furthermore, the company may decide only to offer discrete values of $x$, $y$, and $z$, thus defining a discrete space of customization as shown in Figure 4c.

![Figure 4 -- Spaces of Customization for a Hand Exerciser](image-url)

In the second step, a quantitative criterion that measures the “goodness” of our solution is identified, and an objective function that relates our designs to the value of this criterion is formulated. A typical objective is the minimization of cost, although any other quantitative objective or multiple objectives can be used.

In the third step, various available modes for managing product variety, such as different ways of dimensional customization, adjustable controls, modular combinations, etc. that are applicable to customize the product are identified.

In the fourth step, the modes for managing product variety identified in the third step are organized hierarchically. For example, assume we have three modes for managing product variety to customize a product in some specification $x$. Then, the modes can be organized in a hierarchy like the one shown in Figure 5. If there are $N$ modes for managing product variety, we can use up to $N$ hierarchic levels, one per mode. Note, however, that if fewer levels are used, then more than one mode has to be used in one of the hierarchic levels.

Once a hierarchy is defined in the fourth step, the synthesis of the product platform is then formulated in the fifth step as a multistage optimization problem following the formulation introduced in Section 2. The number of stages of the multi-stage optimization corresponds to the number of hierarchic levels.
Finally, in the sixth step, the optimization problem is solved using an appropriate optimization algorithm. The outcome of this process defines how the various modes for managing product variety are combined to meet custom specifications.

The method is illustrated in the following section with a case example of a custom-made hand exerciser.

4. Case Example: Design of a Custom-Made Hand Exerciser

We illustrate the design of a line of custom-made hand exercisers like the one shown in Figure 6. For brevity we do not discuss the design and manufacturing of the plastic handle and we focus only on the design of the torsion spring. We solve this problem assuming a total demand of 1,000,000 exercisers per year. Please note that this is a simplified example problem. The models used here do not represent those used in the actual design and production of these devices.

![Diagram of a Hand Exerciser](image)

The exerciser is manufactured in the following steps: first, a metallic wire is wound around a mandrel a certain number of turns. After winding, the ends of the wire are cut and a transition bend is made manually between the turns and the legs of the wire. Afterwards, stresses are relieved in an oven. Then, the plastic handles are molded on, the grip squeezed together and a clip placed around the legs to bring the legs to the required initial position.
The exercisers are to be customized in the force $F$ required to close the grip. $F$ is calculated as (Shigley and Mischke, 1996):

$$ F = \frac{d^4E\theta}{68DNR} \quad \text{Eq. (1)} $$

Where $d$ is the diameter of the wire, $E$ is the elastic modulus of the wire material, $D$ is the coil winding diameter, $N$ is the number of coil turns, $R$ is the distance from the coil circle center to the point where the concentrated force $F$ is applied, and $\theta$ is the angular deflection from the no-load position to closing the grip. If we neglect changes in the coil diameter from the free position to the closed position, the angle $\theta$ can be approximated as:

$$ \theta \approx \pi[1 - N + \text{int}(N)] - 2\tan^{-1}\left(\frac{2H}{D}\right) \quad \text{Eq. (2)} $$

$\text{int}(N)$ represents a function that returns the greatest integer $\leq N$. Substituting Equation (2) into Equation (1) yields:

$$ F = \frac{d^4E}{68DNR} \left[\pi[1 - N + \text{int}(N)] - 2\tan^{-1}\left(\frac{2H}{D}\right)\right] \quad \text{Eq. (3)} $$

The stress associated to the force $F$ is approximated as:

$$ \sigma = \frac{32KFR}{\pi d^3} \quad \text{Eq. (4)} $$

where

$$ K = \frac{4k^2 + k - 1}{4k(k - 1)} \quad \text{Eq. (5)} $$

and

$$ k = \frac{D}{d} \quad \text{Eq. (6)} $$

$R$ is considered here constant and approximated as:

$$ R \approx H - \frac{h}{2} \quad \text{Eq. (7)} $$

We design the spring subject to the following geometric and structural constraints:

$$ H \geq h + D \quad \text{Eq. (8)} $$

$$ \sigma \leq S_y \quad \text{Eq. (9)} $$
\[
\sigma \left( \frac{1}{S_e} + \frac{1}{S_u} \right) \leq 1
\]

Eq. (10)

Where \(S_y\), \(S_e\), and \(S_u\) are the yield, endurance and tensile strength of the wire.

The range of force considered in this example is:

\[10 \text{ Newtons} \leq F \leq 100 \text{ Newtons}\]

Eq. (11)

For this example we utilize values of \(w=90 \text{ mm}\) and \(h=100 \text{ mm}\). The design of the springs is carried out for the four scenarios listed in Table 1 and sketched in Figure 7. The second column specifies how many products are included in the space of customization and the third column specifies how we proceed to solve the problem. Finally, the last column indicates how demand is distributed across the space of customization. For example, involves the design of a relatively small number of products, 10 in this case, and we decide to model the space of customization as composed of three discrete points. In this scenario, demand is assumed uniform across the space of customization, i.e., all the 10 forces have the same expected demand. This first scenario is typical of product family design methods.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Increments of Force (F)</th>
<th>Space of Customization</th>
<th>Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10 values</td>
<td>1D - Discrete</td>
<td>Uniform</td>
</tr>
<tr>
<td>2</td>
<td>90 values</td>
<td>1D – Continuous</td>
<td>Uniform</td>
</tr>
<tr>
<td>3</td>
<td>90 values</td>
<td>1D – Continuous</td>
<td>Normal (Gaussian)</td>
</tr>
<tr>
<td>4</td>
<td>90 values</td>
<td>1D – Continuous</td>
<td>Log-normal</td>
</tr>
</tbody>
</table>

Table 1 – Solution Scenarios

Scenario 2 is similar to Scenario 1, but the number of force variants is relatively large, 90 in this case. Finding a solution for such a large number of products in a discrete space of customization is difficult and costly due to the large number of variables involved. We simplify and solve the problem instead by assuming the force \(F\) as continuous and, like in Scenario 1, with a uniform demand across the space of customization.

Scenario 3 is similar to Scenario 2, but we assume a normal (Gaussian) distribution of demand across the space of customization assuming the mean force to be 55 Newtons and its standard deviation, 15. In Scenario 4 we assume that the demand is distributed log-normally with mean value of 30 and standard deviation of 1.5. This distribution shifts the majority of the demand toward the left side of the space of customization as shown below.
4.1 Scenario 1: Solution for Hand Exercisers with 10 Force Increments

Step 1. Define the Space of Customization

The customizable spring is designed for ten different forces: 10, 20, 30, ..., 100 Newtons. As discussed in the preceding section, this is a one-dimensional, discrete space of customization. The total annual demand of products is 1,000,000 and we assume that all 10 forces have the same expected demand (100,000 products per year for each force). Hence, the distribution of demand is uniform.

Step 2. Formulate an Objective

We choose minimization of cost as the objective in this example. Cost is typically formed by the costs of material, labor, tooling, inventory and others. To facilitate the exposition and reproduction of the case example, we assume that most costs can be considered fixed or constant among spring variants and, therefore, do not need to be included in the optimization problem. The cost we consider is as follows:

\[
\text{Cost} = C_{\text{mat}} + C_{\text{tool}} + C_{\text{inv}}
\]

where \(C_{\text{mat}}\) is the cost of the material used in the springs, \(C_{\text{tool}}\) is the cost of the mandrels, and \(C_{\text{inv}}\) the cost of inventory of wire kept in stock, \(C_{\text{inv}}\). All these costs are estimated as total cost in a year.

The annual cost of material is estimated here as:

\[
C_{\text{mat}} = \sum_p \delta(F) C_W \rho \left( \frac{\pi}{4} d^2 L(F, D, W, H) \right)
\]

where \(\delta(F)\) is the force level, \(C_W\) the cost of wire, \(\rho\) the density, and \(L(F, D, W, H)\) the length of the spring.

Now, in Sections 4.1 to 4.3, we apply the six-step method to solve the various scenarios of Table 1.
Where $F$ represents each force of the space of customization, $\delta(F)$ is the annual demand of the spring used with $F$, $C_W$ is the cost in dollars per kg of the required wire, $\rho$ is the density in kg/m$^3$ of the wire, $d$ is the wire diameter and $L(F, D, W, H)$ is the required length of wire used in the spring as a function of the other design variables: $D$, the coil diameter, the wire selection $W$, and $H$, the distance from the bottom of the leg to the coil circle center (see Figure 1). $L(F, D, W, H)$ is approximated as:

$$L(F, D, W, H) = 1.2 \left[\pi DN(F, D, W, H) + 2 \left(\frac{w - D}{2}\right)^2 + H^2\right]^{1/2}$$

Eq. (14)

In Equation (14) there is an extra 20% of wire used to account for the transition bend and other factors. Also, note that $N$ is expressed in Equation (14) not as independent variable but as a function of $F, D, W$ and $H$. This is because we always choose $N$ as the minimum number of turns that satisfies Equation (3) for a given $F$:

$$N(F, D, W, H) = \min\{N(F, D, W, H)\}$$

Eq. (15)

The wires available for selection are shown in Table 2.

<table>
<thead>
<tr>
<th>Wire</th>
<th>$E$ [MPa]</th>
<th>$\rho$ [kg/m$^3$]</th>
<th>$d$ [mm]</th>
<th>$S_y$ [MPa]</th>
<th>$S_u$ [MPa]</th>
<th>$S_e$ [KPa]</th>
<th>$C$ [dollars/kg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>200x10$^3$</td>
<td>7860</td>
<td>0.79</td>
<td>1978</td>
<td>2274</td>
<td>540</td>
<td>3.53</td>
</tr>
<tr>
<td>2</td>
<td>200x10$^3$</td>
<td>7860</td>
<td>1.30</td>
<td>1816</td>
<td>2088</td>
<td>540</td>
<td>2.48</td>
</tr>
<tr>
<td>3</td>
<td>200x10$^3$</td>
<td>7860</td>
<td>1.60</td>
<td>1756</td>
<td>2019</td>
<td>540</td>
<td>2.39</td>
</tr>
<tr>
<td>4</td>
<td>200x10$^3$</td>
<td>7860</td>
<td>1.91</td>
<td>1702</td>
<td>1957</td>
<td>540</td>
<td>2.37</td>
</tr>
<tr>
<td>5</td>
<td>200x10$^3$</td>
<td>7860</td>
<td>2.59</td>
<td>1618</td>
<td>1860</td>
<td>540</td>
<td>2.22</td>
</tr>
<tr>
<td>6</td>
<td>200x10$^3$</td>
<td>7860</td>
<td>3.18</td>
<td>1565</td>
<td>1798</td>
<td>540</td>
<td>2.18</td>
</tr>
<tr>
<td>7</td>
<td>200x10$^3$</td>
<td>7860</td>
<td>4.11</td>
<td>1493</td>
<td>1716</td>
<td>540</td>
<td>2.16</td>
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<td>0.89</td>
<td>1678</td>
<td>1929</td>
<td>540</td>
<td>2.42</td>
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<tr>
<td>9</td>
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<td>1.22</td>
<td>1559</td>
<td>1791</td>
<td>540</td>
<td>2.26</td>
</tr>
<tr>
<td>10</td>
<td>200x10$^3$</td>
<td>7890</td>
<td>1.60</td>
<td>1499</td>
<td>1723</td>
<td>540</td>
<td>2.15</td>
</tr>
<tr>
<td>11</td>
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<td>2.69</td>
<td>1349</td>
<td>1550</td>
<td>540</td>
<td>1.36</td>
</tr>
<tr>
<td>12</td>
<td>200x10$^3$</td>
<td>7890</td>
<td>3.18</td>
<td>1319</td>
<td>1516</td>
<td>540</td>
<td>0.91</td>
</tr>
<tr>
<td>13</td>
<td>200x10$^3$</td>
<td>7890</td>
<td>4.17</td>
<td>1139</td>
<td>1309</td>
<td>540</td>
<td>0.89</td>
</tr>
<tr>
<td>14</td>
<td>200x10$^3$</td>
<td>7890</td>
<td>4.88</td>
<td>1330</td>
<td>1550</td>
<td>540</td>
<td>1.25</td>
</tr>
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<td>15</td>
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<td>5.26</td>
<td>1310</td>
<td>1520</td>
<td>540</td>
<td>1.15</td>
</tr>
</tbody>
</table>

Table 2 -- Available Wires

In order to estimate the cost of tooling (the mandrels), $C_{tool}$, we need to estimate the required number of mandrels $m$. In order to determine $m$ we make the following assumptions:

- A different mandrel is needed for each different coil diameter $D$.
- A mandrel is replaced every $Z$ number of springs, where $Z=50,000$ for this example.
The maximum yield capacity $Y$ per mandrel to maintain a utilization of 90% is 10 springs per hour, and the facility works 16 hours a day. Neglecting down time, $Y=58,400$ springs per year per mandrel.

Based on the aforementioned assumptions, we calculate the annual cost as follows: First, since all springs with the same coil diameter $D$ can share the same mandrel, the total demand for each different diameter $D$ is obtained as the sum of the demand for the various forces with springs with the same coil diameter:

$$\delta_D = \sum_{D(F)} \delta(F)$$

Eq. (16)

Then, the minimum number of mandrels needed at anytime, $m_D$, is the nearest integer higher than the ratio of $\delta_D$ to the yield capacity $Y$ of a mandrel:

$$m_D = \text{int} \left( \frac{\delta_D}{Y} \right) + 1$$

Eq. (17)

The total production per mandrel, $p_D$, is then:

$$p_D = \frac{\delta_D}{m_D}$$

Eq. (18)

Now, let us assume that the cost of a mandrel, $C_{\text{mandrel}}$, is the same for all diameters and equal to 1000 dollars. The cost per year of mandrels used to manufacture coils with diameter $D$, considering the need to replace mandrels every $Z$ number of springs is then:

$$C_{\text{tool}}_D = C_{\text{mandrel}} m_D \frac{p_D}{Z}$$

Eq. (19)

In addition to the cost of Equation (19), we add to the tooling cost an opportunity cost due to the "non-ideal" utilization of equipment. We estimate this tooling opportunity cost as follows:

$$C_\text{tool}_\text{opp} = \sum_D r C_{\text{mandrel}} m_D \left( 1 - \frac{p_D}{Y} \right)$$

Eq. (20)

where $r$ is an annual interest rate (equal to 0.1 for this example). The ratio of $p_D$ to $Y$ yields the percent of time the mandrel is "idle" relative to the "ideal" production $Y$.

The total cost per year of tooling is then:

$$C_{\text{tool}} = \sum_D \left[ C_{\text{mandrel}} m_D \frac{p_D}{Z} + r C_{\text{mandrel}} m_D \left( 1 - \frac{p_D}{Y} \right) \right]$$

Eq. (21)
The only cost from Equation (12) missing is the cost of raw wire inventory, which is calculated assuming that replenishment of wire is made in a “bread-truck” system. In this system, the stock of wires is replenished by the supplier every fixed number of days to a specified level, which we refer as \( I_o \). This level of inventory \( I_o \) is a variable that needs to be determined. We estimate this variable assuming that we want the minimum level at the beginning of the replenishment period that yields a 99% statistical confidence that no shortage of wire will be incurred between replenishments.

Let us state that the replenishment occurs once a week and that the demand of the custom-made exercisers is a Poisson random process. It follows the demand of the wire used by any of these exercisers is also a Poisson process. Let say that the demand rate in Tons of wire per week for the wire \( W \) is \( \lambda_W \). Let \( \lambda \) represent the Tons of wire used from the beginning to the end of the time between replenishments and \( I_{ow} \) the value in Tons of wire for which we satisfy the stated requirement of having a service level of 99%, i.e., the value for which \( P\{X \leq I_{ow}\} \geq 0.99 \). Since the process is Poisson, this is written as:

\[
e^{-\lambda_W} \sum_{x=0}^{I_{ow}} \frac{\lambda_W^x}{x!} \geq 0.99
\]

Eq. (22)

Hence, given a selection of wires for the various springs in the space of customization, we obtain the minimum value \( I_{ow} \) that satisfies Eq. (22) and then we approximate the average inventory as:

\[
I_{w} = I_{ow} - E(X) = I_0 - \frac{\lambda_W t}{2}
\]

Eq. (23)

where \( t \) is the time between replenishments (1 week).

Also, we add to the cost of inventory an extra overhead cost due to the additional purchase processing, warehouse space and other factors associated with each wire used. Let us say that the extra cost is \( K_W \) for each wire used. We do this to examine how our solution changes as the extra cost due to the wires variety changes. Hence, using \( I_{w} \) from Equation (23) and \( K_W \), the total inventory cost per year is then:

\[
C_{inv} = \sum_W (rC_W I_w + K_W)
\]

Eq. (24)

where \( C_W \) is the cost of wire per kg from Table 2. We solve the problem for two different values of \( K_W \): \( K_W=0 \) (no extra overhead per wire) and \( K_W=2000 \) dollars per wire per year.

The total cost per year is then calculated by substituting Equations (13), (21) and (24) in Equation (12). We proceed now to the third step of the method.

**Step 3. Identify Modes for Managing Product Variety**

Customization of the exercisers is achieved through the following modes for managing product variety:
1. Varying the distance $H$
2. Varying the number of turns $N$
3. Varying the coil diameter $D$
4. Varying the wire selection (i.e., the wire diameter $d$ and its material properties)

Now we proceed to Step 4 to determine how many hierarchic levels to utilize and which modes should be used in which hierarchy level.

**Step 4. Determine Number of Hierarchy Levels and Allocate the Modes for Managing Product Variety to These Levels**

In this example, let us use three hierarchic levels. We refer to the smallest divisions as first-space divisions, $\Delta F_1$; the assembly of these as second-space divisions, $\Delta F_2$; and, finally, the assembly of the later as third-space division, $\Delta F_3$.

**Step 4.1 First Space Division**

In order to allocate the modes for managing product variety to $\Delta F_1$, the smallest space division, we look at the third of the principles listed in Section 2. This principle tell us that the most flexible and economical modes, i.e., those that deal best with high frequency and short-scope changes, should be used at the lower levels. For products that have adjustable controls or features (e.g., adjusting temperature of an oven or the height of a bicycle seat), we typically utilize these controls at this lowest level. In the case of the exercisers, we do not have any adjustable controls or features and customization is therefore fixed at manufacturing. In this case, note that for a given $w$ and $h$, it is economical and easy to vary the number of turns $N$ and the distance $H$ since no additional tooling or operations are required. In this case we decide to use a common value of $H$ between all products within each first space division and then adjusting the number of turns to satisfy the required customization force of each exerciser contained in the space division.

**Step 4.2 Second Space Division**

The second space division is formed as an assembly of first-space divisions, each with its own value of $H$. In this example, it is not obvious which of the two remaining available modes is more flexible and economical. Varying the number of wires involves more inventory whereas varying the number of coil diameters involves more tooling. In cases like this, where the allocation of modes to a hierarchy level is difficult, it is necessary to solve the problem for one ordering and look at the solution: if the size of the space element at the lower level is the same as the one of the higher level, then we should reverse the allocation of the modes between these two levels and solve again the problem. This is shown later in the case example. At this point, let us allocate varying the wire selection in the second hierarchy level. Hence, each second space division will have its own independent wire selection and will contain a number of first space divisions, each in turn with its own value of $H$.

**Step 4.3 Third Space Division**
The only remaining mode for managing product variety available is varying the coil diameter, which we utilize at the highest hierarchy level, which is therefore allocated to the third space division, $\Delta F_3$.

In summary, the space of customization is “accessed” through a hierarchic construct of modes for managing product variety as follows:
1. Varying the value of $D$ at the third (largest) space divisions that contain a number of second space divisions.
2. Varying the wire selection at the second space divisions, which in turn contain a number of first space divisions.
3. Varying the height $H$ at the first (smallest) space divisions.
4. Adjusting the number of coils for any specified force given the values of $D$, $W$ and $H$.

A sketch of the hierarchic organization of modes for managing product variety is shown in Figure 8. Using this construct we proceed to formulate our problem mathematically in Step 5.

![Hierarchic Organization of the Modes for Managing Product Variety for Hand Exercisers](image)

**Step 5. Formulate a Multi-Stage Optimization Problem**

In order to formulate our optimization problem for this example, we apply the sub-indices $i$, $j$ and $k$ to the variables associated to the first, second and third space element respectively. Hence, the coil diameter used in the $k$ 3rd space division is referred as $D_k$. Similarly, the wire used in the $j$ 2nd space division contained in the $k$ 3rd space division is $W_{jk}$. Finally, $H_{ijk}$ is the value of $H$ used in the $i$ 1st space division contained in the $j$ 2nd space and $k$ 3rd space. Using this nomenclature we have the following decision variables:

- The size of the 3rd space divisions, $\Delta F_{3,k}$, and the value of $D_k$ for each of these spaces.
- The size of the 2nd space divisions, $\Delta F_{2,j,k}$, in which we divide each $k$ 3rd space and the value of the wire $W_{jk}$ for each of these spaces.
- The size of the 1st space divisions, $\Delta F_{1,i,j,k}$, in which we divide each of the 2nd space divisions and the value $H_{ijk}$ for each of these.

Using this nomenclature, we write the number of turns for a force $F$ contained in the $ijk$ 1st space element as:

$$N_{ijk}(F) = N(F, D_k, W_{jk}, H_{ijk})$$

Eq. (25)
In addition to the product design constraints, Equations (8) to (10), we add three additional constraints in the formulation of the problem. First, we bound the minimum size of any space element to be one tenth of the whole range of customization. Hence, since the entire range of force is 90 Newtons, all spaces are constrained to be larger than 9 Newtons. These bounds and the requirement of having a hierarchic solutions yield the following six constraints:

\[ 9 \leq \Delta F_{3k} \leq 90 \]  
\[ \sum_k \Delta F_{3k} = 90 \]  
\[ 9 \leq \Delta F_{2jk} \leq \Delta F_{3k} \]  
\[ \sum_j \Delta F_{2jk} = \Delta F_{3k} \]  
\[ 9 \leq \Delta F_{1ijk} \leq \Delta F_{2jk} \]  
\[ \sum_i \Delta F_{1ijk} = \Delta F_{2jk} \]

Now, let us formulate the objective function for each stage. First, we write the total cost defined in Equation (12) as a function of all decision variables:

\[ \text{Cost} = \text{Cost}(\Delta F_{1ijk}, H_{ijk}, \Delta F_{2jk}, W_{jk}, \Delta F_{3k}, D_k) \]  

We write the minimum cost as:

\[ \text{Cost}^* = \min \left\{ \text{Cost}(\Delta F_{1ijk}, H_{ijk}, \Delta F_{2jk}, W_{jk}, \Delta F_{3k}, D_k) \right\} \]  

Based on the principle of optimality (Winston, 1994) we rewrite Equation (33) as a three-stage minimization problem as follows:

\[ \text{Cost}^* = \min \left\{ \min \left\{ \min \left\{ \text{Cost}(\Delta F_{1ijk}, H_{ijk}, \Delta F_{2jk}, W_{jk}, \Delta F_{3k}, D_k) \right\} \right\} \right\} \]  

As we discussed in Section 2, one desirable property of a hierarchic organization is the so-called near-decomposability. We can obtain this property by synthesizing the branches of the hierarchy independently of other branches. For our case example, this implies that the selection of a design variable for a space element should not be affected by the selection of the design variables in the other spaces at the same level. In other words, the selection of \( D \) for a particular 3rd space element should be independent of the selection of \( D \) for all other 3rd space elements. Same requirement applies to the selection of \( W \) for each 2nd space elements and \( H \) for the 1st space elements. If it is better from a total cost perspective to have a common variable across two contiguous space elements, this should be captured in the solution not by selecting the same wire for the two spaces but by having one larger space element spanning both neighboring spaces. We capture this
notion by reformulating the internal cost in Equation (34) as a vector of costs through the sub-indexes \( ijk \):

\[
Cost^* = \min \{( \min \{( \min \{ Cost_{ijk}(\Delta F_{ijk}, H_{ijk}, \Delta F_{2,jk}, W_{jk}, \Delta F_{3,k}, D_k) \})\})\} \quad \text{Eq. (35)}
\]

Let us refer to the interior minimization term in Equation (34) as:

\[
\min_{\Delta F_{ijk}, H_{ijk}} \{ Cost_{ijk}(\Delta F_{ijk}, H_{ijk}, \Delta F_{2,jk}, W_{jk}, \Delta F_{3,k}, D_k) \} \equiv Cost^{(2)}_{jk}(\Delta F_{2,jk}, W_{jk}, \Delta F_{3,k}, D_k)
\]

Eq. (36)

And then we rewrite Equation (34) as:

\[
Cost^* = \min \{( \min \{ Cost^{(2)}_{jk}(\Delta F_{2,jk}, W_{jk}, \Delta F_{3,k}, D_k) \})\} \quad \text{Eq. (37)}
\]

Similarly, let us refer to the interior minimization term in Equation (37) as:

\[
\min_{\Delta F_{2,jk}, W_{jk}} \{ Cost^{(2)}_{jk}(\Delta F_{2,jk}, W_{jk}, \Delta F_{3,k}, D_k) \} \equiv Cost^{(3)}_{k}(\Delta F_{3,k}, D_k)
\]

Eq. (38)

And substituting Equation (38) in (37) yields:

\[
Cost^* = \min \{ Cost^{(3)}_{k}(\Delta F_{3,k}, D_k) \} \quad \text{Eq. (39)}
\]

With Equations (26) to (39) we proceed to formulate the baseline optimization for each stage follows.

<table>
<thead>
<tr>
<th><strong>Baseline Optimization Problem for the 1st Stage</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Given</strong></td>
</tr>
<tr>
<td>( w=90 ) mm, ( h=100 ) mm</td>
</tr>
<tr>
<td>The 2nd space divisions ( \Delta F_{2,jk} )</td>
</tr>
<tr>
<td>The wires ( W_{jk} ) and coil diameters ( D_k )</td>
</tr>
<tr>
<td><strong>Find</strong></td>
</tr>
<tr>
<td>The size of the 1st space divisions ( \Delta F_{1,ijk} )</td>
</tr>
<tr>
<td>The value ( H_{ijk} ) for each 1st space division</td>
</tr>
<tr>
<td><strong>Satisfy</strong></td>
</tr>
<tr>
<td>Product Constraints, Eqs. (8) to (10), for all forces ( F \in \Delta F_{1,ijk} )</td>
</tr>
<tr>
<td>9 ( \leq \Delta F_{1,ijk} \leq \Delta F_{2,jk} ) \quad \text{Eq (30)}</td>
</tr>
<tr>
<td>( \sum\Delta F_{1,ijk} = \Delta F_{2,jk} ) \quad \text{Eq (31)}</td>
</tr>
<tr>
<td><strong>Minimize</strong></td>
</tr>
<tr>
<td>( Cost_{ijk}(\Delta F_{1,ijk}, H_{ijk}, \Delta F_{2,jk}, W_{jk}, \Delta F_{3,k}, D_k) ) for each 1st space ( ijk )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Baseline Optimization Problem for the 2nd Stage</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Given</strong></td>
</tr>
<tr>
<td>The ( k ) 3rd space divisions ( \Delta F_{3,k} )</td>
</tr>
<tr>
<td>The coil diameter ( D_k )</td>
</tr>
</tbody>
</table>
Find The size of the 2nd space division $\Delta F_{2jk}$
The wire for each 2nd space division $W_{jk}$

Satisfy Product Constraints, Eqs. (8) to (10), for all forces $F \in \Delta F_{2jk}$

\[ 9 \leq \Delta F_{2jk} \leq \Delta F_{3k} \quad \text{Eq (27)} \]
\[ \sum_j \Delta F_{2jk} = \Delta F_{3k} \quad \text{Eq (28)} \]

Minimize $Cost^{(2)}_{jk}(\Delta F_{2jk}, W_{jk}, \Delta F_{3k}, D_k)$ for each 2nd space $jk$

---

**Baseline Optimization Problem for the 3rd Stage**

Find The size of the 3rd space divisions $\Delta F_{3k}$
The coil diameter for each 3rd space division $D_k$

Satisfy Product Constraints, Eqs. (8) to (10), for all forces $F \in \Delta F_{3k}$

\[ 9 \leq \Delta F_{3k} \leq 90 \quad \text{Eq (25)} \]
\[ \sum_k \Delta F_{3k} = 90 \quad \text{Eq (26)} \]

Minimize $Cost^{(3)}_{k}(\Delta F_{3k}, D_k)$ for each 3rd space $k$

And we integrate the baseline decisions into a single multi-stage optimization problem. The resulting formulation follows:

Given $w=90$ mm, $h=100$ mm

Find The size of the 3rd, 2nd and 1st space divisions: $\Delta F_{3k}, \Delta F_{2jk}$ and $\Delta F_{1ijk}$
The coil diameter for each 3rd space division $D_k$
The wire for each 2nd space division $W_{jk}$
The value $H_{ijk}$ for each 1st space division

Satisfy Product Constraints, Eqs. (8) to (10), for all forces $F$

\[ 9 \leq \Delta F_{3k} \leq 90 \quad \text{Eq (29)} \]
\[ \sum_k \Delta F_{3k} = 90 \quad \text{Eq (30)} \]
\[ 9 \leq \Delta F_{2jk} \leq \Delta F_{3k} \quad \text{Eq (31)} \]
\[ \sum_j \Delta F_{2jk} = \Delta F_{3k} \quad \text{Eq (32)} \]
\[ 9 \leq \Delta F_{1ijk} \leq \Delta F_{2jk} \quad \text{Eq (33)} \]
\[ \sum_i \Delta F_{1ijk} = \Delta F_{2jk} \quad \text{Eq (34)} \]

Minimize $Cost(\Delta F_{1ijk}, H_{ijk}, \Delta F_{2jk}, W_{jk}, \Delta F_{3k}, D_k)$
Next, in Step 6, we solve this multi-stage optimization problem.

**Step 6. Solve the Optimization Problem**

The multi-stage optimization problem formulated in Step 5 is solved through an exhaustive search on all feasible combinations of $\Delta F_1$, $\Delta F_2$ and $\Delta F_3$. The spring variables are found using an appropriate optimization technique at each level of a nested optimization process (see Figure 9). In this case, the values of $D$ and $H$ are found through a Golden-Section searches with penalty constraint functions (Winston, 1994). The selection of wire is found with a Branch-and-Bound algorithm (Winston, 1994).

![Figure 9 – Solution Algorithm for the Multi-Stage Optimization Problem](image)

**Results Scenario 1**

The resulting division of the space of customization for this first scenario is shown in Figure 10 for the overhead cost from Equation (26) $K_W=0$ and $K_W=2000$ respectively. The number inside each space is the value of the corresponding $D$, $W$ or $H$ for that particular space. The costs of the two solutions are shown in Figure 11. Note that in both solutions, the size of the smallest divisions is as small as allowed by the constraint $9 \leq \Delta F_{1ijk} \leq \Delta F_{2jk}$. This is expected since we have not included in our cost function any cost of variety associated with multiple values of $H$. Therefore, cost is minimized by optimizing the value of $H$ for each force, thus resulting in as many 1st space elements as allowed. For the case with $K_W=0$, the minimum cost was obtained using 3 coil diameters and 4 different wires. However, when $K_W=2000$, the cost of inventory becomes significant and the number of wires is then reduced from 4 to 2 to minimize the total cost.

\[
K_W=0 \quad \quad K_W=2000
\]
Observe that when $K_W=2000$, the size of the 2nd space divisions is as large as the size of the divisions above them. This is an indication that a better solution is likely to be obtained by reversing the allocation of design variables between the corresponding 2nd and 3rd levels. The results of this reversal are shown in Figure 12 and 13. As expected, we obtain a lower cost. Note that in this case the reversal does not affect the amount of variety of wires, which is still 4 and 2 respectively. More commonality of wires (one wire) is not possible due to the product stress constraints. However, the solution yields a larger number of coil diameters. Even though this increases the tooling cost, the reversal allowed a further reduction in material cost that offsets the extra tooling cost.
4.2 Scenarios 2 to 4: Solution for Hand Exercisers with 90 Force Increments

The process to solve Scenarios 2 to 4 is very similar to the one described for Scenario 1 in Section 4.1. The only difference is in the first two steps of the method.

Step 1. Define the Space of Customization

The customizable spring is designed for 90 different forces: 10, 11, 12, ..., 100 Newtons. As discussed previously, solving this problem as a discrete one is costly and difficult. Instead we assume that the space of customization is continuous in the range of 10 to 100 Newtons. The total annual demand of products is again 1,000,000. The distribution of demand for three scenarios, illustrated previously in Figure 7, is:

- Scenario 2: Demand is uniform: all forces have the same probability of being ordered at any time:

  $$p(F) = \frac{1}{100 - 10}$$

  Eq. (40)
• Scenario 3: The probability of a particular force in the space of customization being ordered at any time is assumed normal, with mean $\mu = 55$ and standard deviation of $\sigma = 15$:

$$p(F) = \frac{1}{\sqrt{2\pi} \cdot \sigma} \exp \left[ -\frac{1}{2} \left( \frac{F - \mu}{\sigma} \right)^2 \right]$$  \hspace{1cm} \text{Eq. (41)}

• Scenario 4: The probability of a particular force in the space of customization being ordered at any time is assumed log-normal, with log-mean $\mu_{ln} = \ln(30)$ and log-standard deviation $\sigma_{ln} = \ln(1.5)$:

$$p(F) = \frac{1}{\sqrt{2\pi} \cdot \sigma_{ln} \cdot F} \exp \left[ -\frac{1}{2} \left( \frac{\ln(F) - \mu_{ln}}{\sigma_{ln}} \right)^2 \right]$$  \hspace{1cm} \text{Eq. (42)}

And, given the total annual estimated demand of 1,000,000 springs, the demand for a particular value of $F$ is then estimated as:

$$\delta(F) = 1 \times 10^6 \cdot p(F)$$  \hspace{1cm} \text{Eq. (43)}

Step 2. Formulate an Objective

In this case, we cannot add the cost of material for each spring, since we are assuming a continuous space. Instead, we simply substitute the summation in Equation (13) with an integral:

$$Cmat = \int_{10}^{100} \delta(F) \cdot C_w \cdot \rho \left( \frac{\pi}{4} d^2 L(F,D,W,H) \right) dF$$  \hspace{1cm} \text{Eq. (43)}

This cost can be rewritten as the summation of the cost of material of all the 1st space elements:

$$Cmat = \frac{\pi}{4} \sum_k \sum_{j<k} C_{wjk} \rho_{wjk} d_{wjk}^2 \sum_{i<j<k}^{F_{ijk} \text{max}} \sum_{i<j<k}^{F_{ijk} \text{min}} \delta(F) L_{ijk}(F) dF$$  \hspace{1cm} \text{Eq. (44)}

Where $F_{ijk} \text{min}$ and $F_{ijk} \text{max}$ are the minimum and maximum values of force of each 1st space element and $L_{ijk}(F)$ is obtained by substituting $D_i$, $W_{jk}$ and $H_{ijk}$ in Equation (14).

Since the number of coil diameters and wires is still limited to the finite number of 3rd and 2nd spaces, the approach to estimate the cost of tooling and inventory remains unchanged and, therefore, our cost is estimated by using Equation (43) in Equation (12).

In this case, it is difficult to obtain a closed-form equation to the integration of Equation (44). Therefore, this integration is carried out numerically using numerical integration during the optimization process. Since Steps 3 to 6 discussed in Section 4.1 are not affected by our re-formulation of the problem, we proceed to discuss the results of Scenarios 2 to 4.
Results Scenarios 2 through 4

The resulting division of the space of customization for the three scenarios and both $K_W = 0$ and $K_W = 2000$ are shown in Figure 14 to 16. Observe that in the case of a uniform distributed demand, as in Scenario 1, the effect of adding the extra overhead cost $K_W = 2000$ is to cause the 2nd spaces to be as large as the 3rd spaces above them. As in Scenario 1, we are likely to obtain a better solution by reversing the utilization of $D$ and $W$ between these two hierarchic levels.

In the case of the normal distribution of demand, we observe that the change from $K_W = 0$ to 2000 did not affect the solution. On the other hand, the effect of this change was significant for the log-normally distributed demand. This is because in the later case, there is relatively little demand for the forces on the right-tail of the space of customization (see Figure 7) and, consequently, optimization of the material cost for the larger forces has lesser weight that in the other two scenarios. Lower costs are obtained instead by reducing the variety in wires and coil diameters. Note that, as in Scenario 1, the 2nd space divisions tend to be as large as the 3rd space divisions above them. This again, indicates that a better solution will probably be obtained by reversing the utilization of $D$ and $W$ in these two hierarchic levels.

Figure 14 -- Solution for Scenario 2: Uniformly Distributed Demand

<table>
<thead>
<tr>
<th>$K_W = 0$</th>
<th>$K_W = 2000$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D$ [mm]</td>
<td>167</td>
</tr>
<tr>
<td>$W$ [mm]</td>
<td>11</td>
</tr>
<tr>
<td>$H$ [mm]</td>
<td>168</td>
</tr>
<tr>
<td>Force [Newtons]</td>
<td>10</td>
</tr>
</tbody>
</table>

Figure 15 -- Solution for Scenario 3: Normally Distributed Demand

<table>
<thead>
<tr>
<th>$K_W = 0$</th>
<th>$K_W = 2000$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D$ [mm]</td>
<td>350</td>
</tr>
<tr>
<td>$W$ [mm]</td>
<td>11</td>
</tr>
<tr>
<td>$H$ [mm]</td>
<td>166</td>
</tr>
<tr>
<td>Force [Newtons]</td>
<td>10</td>
</tr>
</tbody>
</table>
Figure 16 -- Solution for Scenario 4: Lognormally Distributed Demand

<table>
<thead>
<tr>
<th>$D$ [mm]</th>
<th>217</th>
<th>261</th>
<th>167</th>
<th>342</th>
<th>203</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>Wire</em></td>
<td>11</td>
<td>12</td>
<td>12</td>
<td>13</td>
<td>13</td>
</tr>
<tr>
<td>$H$ [mm]</td>
<td>157</td>
<td>136</td>
<td>152</td>
<td>134</td>
<td>117</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$D$ [mm]</th>
<th>186</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>Wire</em></td>
<td>12</td>
</tr>
<tr>
<td>$H$ [mm]</td>
<td>191</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$D$ [mm]</th>
<th>120</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>Wire</em></td>
<td>13</td>
</tr>
<tr>
<td>$H$ [mm]</td>
<td>160</td>
</tr>
</tbody>
</table>
From these results, we see that how much commonality is desirable in the line of customizable exercisers is affected significantly by how we model the various costs involved and our assumptions of the demand across products.

In summary we have shown in this case example that the proposed method is an effective approach to product design for mass customization since it supports designers to combine systematically and rigorously multiple approaches for product customization. It can be applied to small or large number of products and can be solved for any distribution of demand among the parameters to be customized and it allows to test assumptions and answer what-if questions by testing the effect of different distributions of demand and/or cost models.

5. Closure

In this paper we presented a product design method that enables us to combine systematically multiple approaches for product customization. The method exploits the advantages of modeling the design of customizable products as a problem of optimization of access in a geometric space, and then solving the resulting access problem through the hierarchic organization of approaches for product customization.

One difficulty of this method is the need to formulate explicitly an objective function that captures the various costs involved in a product design. This method, like any other product family design method, will typically yield better results when more complete models of cost are utilized. Accurate and complete cost models can, however, be difficult to obtain and/or solve, particularly for new product lines, when little knowledge or experience is available.

Among the advantages of this method are:
- Adaptability. The method enforces the property of near-decomposability of hierarchic systems. This property allow the alteration of product designs for portions of the
space of customization without affecting other products of the family that are not in the same branch of the hierarchic construct.

- **Cost-Effectiveness.** The method offers a rigorous approach for balancing effectively the tradeoff between the various costs involved in product customization.
- **Suitable for small or large variety in the product specifications.** The method can be applied to a small number of product variants by formulating the space of customization as a discrete one or to a large number of variants -as is typically the case in mass customization- by formulating the space of customization as a continuous one.
- **Applicable to any distribution of demand.** The method, as shown in the example problem, can be applied to any distribution of demand in the space of customization. This allows us not only to capture more accurately the tradeoff between costs and tune our product design to our most profitable portions of the market, but also to study the robustness of our solution to changes in the market.

Although the example presented in this paper has not shown this element of the method, it is applicable to multiple customizable specifications (discrete, continuous or mixed). Examples of this type of problems can be found in (Hernandez, 2001) and (Hernandez, et al., 2002).

**References**


